

Is Neutrino a Superluminal Particle?

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Abstract

Based on the experimental discovery that the mass-square of neutrino is negative, a quantum theory for superluminal neutrino is proposed. Two Weyl equations coupled together via a mass term respecting the maximum parity violation lead to a new equation which describes the superluminal motion of neutrino with permanent helicity. Various strange features of subluminal and superluminal particles can be ascribed to the relative variation of two contradictory fields superposing coherently inside the particle with the change of its speed u in the whole range ($0 < u < \infty$). Being compatible with the theory of special relativity, this theory may have various applications.

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Recent experimental data in the measurement of tritium beta decay reveal an amazing result that the mass-square of electron neutrino is negative [1]:

$$m^2(\nu_e) = -2.5 \pm 3.3 eV^2. \quad (1)$$

The pion decay experiment also shows a similar puzzle that [1]:

$$m^2(\nu_\mu) = -0.016 \pm 0.023 MeV^2. \quad (2)$$

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The experimental data (1) and (2), though far from accurate, strongly hint that a neutrino might be some particle moving faster than light. Following the existing literature [2,3,4,5,6], we shall call it superluminal particle or tachyon, which obeys the kinematic relation:

$$E^2 = c^2 p^2 - m_s^2 c^4 \quad (3)$$

with its energy, momentum and “proper mass” denoted by E , p and m_s respectively, For instance $m_s(\nu_e) = 1.6eV$. In a short letter [7] a new Dirac-type equation is established for explaining the negative mass-square of neutrino. As a continuation and development of [7], here we will elaborate the quantum theory in some detail and explore further the intrinsic essence responsible for the strange behavior of tachyon.

Since the discovery of parity violation in 1956 [8, 9], the theory for neutrino is based on Weyl equation

$$i\hbar \frac{\partial}{\partial t} \xi = i\hbar \vec{\sigma} \cdot \nabla \xi, \quad (4)$$

where $\xi(t, x)$ is a two-component spinor function. Eq. (4) describes a positive energy ($E > 0$) solution for left-handed neutrino (with helicity $H = \langle \vec{\sigma} \cdot \hat{\vec{p}} \rangle = -1$, $\hat{\vec{p}} = \vec{p}/|\vec{p}|$) and a negative energy ($E < 0$) solution for right-handed antineutrino (with helicity $H = 1$) in accordance with that verified by experiments. The alternative possibility that

$$i\hbar \frac{\partial}{\partial t} \eta = -i\hbar \vec{\sigma} \cdot \nabla \eta, \quad (5)$$

was thus abandoned. As now experiments show that $m_s \neq 0$, we assume a new equation for neutrino being composed of both ξ and η coupling via nonzero m_s :

$$\begin{aligned} i\hbar \frac{\partial}{\partial t} \xi &= i\hbar \vec{\sigma} \cdot \nabla \xi - m_s c^2 \eta, \\ i\hbar \frac{\partial}{\partial t} \eta &= -i\hbar \vec{\sigma} \cdot \nabla \eta + m_s c^2 \xi \end{aligned} \quad (6)$$

Consider a plane wave solution $\xi \sim \eta \sim \exp[i(px - Et)/\hbar]$ along x axis for a particle with helicity $H = -1$, we find

$$\eta = \frac{m_s c^2}{cp + E} \xi \quad (7)$$

and Eq.(3) as expected. Based on Eq.(3) with quantum relations $E = \hbar\omega$ and $p = \hbar k$, the velocity of particle, u , should be identified with the group velocity $u_g = \frac{d\omega}{dk}$ of wave versus the phase velocity $u_p = \frac{\omega}{k}$. Defining the changeable (total) mass \tilde{m} by $p = \tilde{m}u_g = \tilde{m}u$, one can easily prove that:

$$u_p u_g = c^2, \quad (8)$$

$$p = \tilde{m}u = \frac{m_s u}{\sqrt{\frac{u^2}{c^2} - 1}}, \quad E = \tilde{m}c^2 = \frac{m_s c^2}{\sqrt{\frac{u^2}{c^2} - 1}}. \quad (9)$$

For understanding why a negative solution ($E < 0$) describes an antiparticle, we introduce two linear combination functions of ξ and η :

$$\varphi = \frac{1}{\sqrt{2}}(\xi + \eta), \quad \chi = \frac{1}{\sqrt{2}}(\xi - \eta) \quad (10)$$

$$\begin{aligned} i\hbar \frac{\partial}{\partial t} \varphi &= i\hbar \vec{\sigma} \cdot \nabla \chi + m_s c^2 \chi, \\ i\hbar \frac{\partial}{\partial t} \chi &= i\hbar \vec{\sigma} \cdot \nabla \varphi - m_s c^2 \varphi \end{aligned} \quad (11)$$

Eq. (11) is invariant under the space-time inversion ($\vec{x} \longrightarrow -\vec{x}$, $t \longrightarrow -t$) and transformation:

$$\varphi(-\vec{x}, -t) \longrightarrow \chi(\vec{x}, t), \quad \chi(-\vec{x}, -t) \longrightarrow \varphi(\vec{x}, t) \quad (12)$$

(see Refs. [10,11]). For a concrete solution of particle:

$$\varphi \sim \chi \sim \exp[i(px - Et)/\hbar], \quad (E > 0) \quad (13)$$

$$\chi = \frac{cp - m_s c^2}{E} \varphi, \quad (|\varphi/\chi| > 1) \quad (14)$$

the transformation (12) leads to the wavefunction (WF) of its antiparticle:

$$\begin{aligned} \varphi(-\vec{x}, -t) &\longrightarrow \chi_c(\vec{x}, t) \sim \exp[-i(px - Et)/\hbar], \\ \chi(-\vec{x}, -t) &\longrightarrow \varphi_c(\vec{x}, t) \sim \exp[-i(px - Et)/\hbar], \end{aligned} \quad (|\chi_c/\varphi_c| > 1) \quad (15)$$

Eq.(15) implies a negative energy solution if we use the familiar operators of momentum and energy for particle:

$$\hat{p} = -i\hbar \frac{\partial}{\partial x}, \quad \hat{E} = i\hbar \frac{\partial}{\partial t} \quad (16)$$

But for antiparticle, we should use the counterparts of (16) as [11]

$$\hat{p}_c = i\hbar \frac{\partial}{\partial x}, \quad \hat{E}_c = -i\hbar \frac{\partial}{\partial t} \quad (16)$$

(subscript c denotes the antiparticle). Hence Eq. (15) describes an antiparticle with momentum p and energy $E(> 0)$ precisely as that in Eq. (13) for a particle. Interesting enough, a modification of Eq. (11) as

$$\begin{aligned} i\hbar \frac{\partial}{\partial t} \varphi_D &= i\hbar \vec{\sigma} \cdot \nabla \chi_D + m_0 c^2 \varphi_D, \\ i\hbar \frac{\partial}{\partial t} \chi_D &= i\hbar \vec{\sigma} \cdot \nabla \varphi_D - m_0 c^2 \chi_D \end{aligned} \quad (18)$$

which still obeys the symmetry (12) is just the Dirac equation:

$$i\hbar \frac{\partial}{\partial t} \psi_D = i\hbar \vec{\alpha} \cdot \nabla \psi_D + \beta m_0 c^2 \psi_D \quad (19)$$

$$\psi_D = \begin{pmatrix} \varphi_D \\ \chi_D \end{pmatrix}, \quad \alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad (20)$$

for describing the electron with rest mass m_0 and speed $u < c$.

Why Eq.(11) is so radically different from the Eq.(18)? Let's first derive the continuity equation for Eq. (11) (or Eq.(6)):

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{j} = 0, \quad (21)$$

$$\rho = \varphi^+ \chi + \chi^+ \varphi = \xi^+ \xi - \eta^+ \eta, \quad (22)$$

$$\vec{j} = -c(\varphi^+ \vec{\sigma} \varphi + \chi^+ \sigma \chi) = -c(\xi^+ \vec{\sigma} \xi + \eta^+ \vec{\sigma} \eta), \quad (23)$$

versus that for Dirac Eq.(18) or its equivalent form:

$$\xi_D = \frac{1}{\sqrt{2}}(\varphi_D + \chi_D), \quad \eta_D = \frac{1}{\sqrt{2}}(\varphi_D - \chi_D) \quad (24)$$

$$i\hbar \frac{\partial}{\partial t} \xi_D = i\hbar \vec{\sigma} \cdot \nabla \xi_D + m_0 c^2 \eta_D, \quad (25)$$

$$i\hbar \frac{\partial}{\partial t} \eta_D = -i\hbar \vec{\sigma} \cdot \nabla \eta_D + m_0 c^2 \xi_D$$

$$\rho_D = \varphi_D^+ \varphi_D + \chi_D^+ \chi_D = \xi_D^+ \xi_D + \eta_D^+ \eta_D, \quad (26)$$

$$\vec{j}_D = -c(\varphi_D^+ \vec{\sigma} \chi_D + \chi_D^+ \vec{\sigma} \varphi_D) = -c(\xi_D^+ \vec{\sigma} \xi - \eta_D^+ \vec{\sigma} \eta_D). \quad (27)$$

Now the normalization condition $\int \rho d\vec{x} = 1$ for Eq.(6) corresponds to the conservation of helicity in the motion: $H = -1$ for particle whereas $H = 1$ for antiparticle. The neutrino (antineutrino) is permanently longitudinal polarized as $\nu_L(\bar{\nu}_R)$ and its invariant feature can be maintained in any inertial frame because of its velocity $u > c$.

Next, we can find out a radical difference between (6) and (25). Under the space-inversion ($\vec{x} \longrightarrow -\vec{x}$) and related transformation:

$$\xi_D(-\vec{x}, t) \longrightarrow \eta_D(\vec{x}, t), \quad \eta_D(-\vec{x}, t) \longrightarrow \xi_D(\vec{x}, t) \quad (28)$$

Dirac Eq.(25) is invariant whereas Eq. (6) fails to do so because of the opposite sign in mass term. It is just a clearcut reflection of the fact that neutrino yields the maximum

violation of parity. The new observation is that the parity violation is triggered by its nonzero (proper) mass which in turn implies that neutrino must be a superluminal particle with permanent helicity: while ν_L and $\bar{\nu}_R$ are allowed, ν_R and $\bar{\nu}_L$ must be forbidden strictly.

We are now in a position to realize the marvelous kinematical feature of superluminal particle together with that of subluminal one. We define the “ratio” R of “hidden amplitude of antiparticle state” to that of “particle state” in a particle (as what had been done for Dirac particle in Ref. [12] with R there being redefined as R^2 here):

$$R \equiv \sqrt{\frac{\chi^+ \chi}{\varphi^+ \varphi}} = \left[\frac{\frac{u}{c} - \sqrt{\frac{u^2}{c^2} - 1}}{\frac{u}{c} + \sqrt{\frac{u^2}{c^2} - 1}} \right]^{\frac{1}{2}}, \quad (u > c) \quad (29)$$

$$R = \left[\frac{1 - \sqrt{1 - \frac{u^2}{c^2}}}{1 + \sqrt{1 - \frac{u^2}{c^2}}} \right]^{\frac{1}{2}}, \quad (u < c). \quad (30)$$

Similarly, we define a “Weyl parameter” W as the ratio of “hidden amplitude of right-handed helicity state” to that of “left-handed helicity state” in a particle with helicity $H = -1$:

$$W = \sqrt{\frac{\eta^+ \eta}{\xi^+ \xi}} = \sqrt{\frac{u - c}{u + c}}, \quad (u > c) \quad (31)$$

$$W = \sqrt{\frac{c - u}{c + u}}, \quad (u < c). \quad (32)$$

Being functions of (u/c) , the values of R and W are symmetric with respect to $u/c = 1$ in logarithmic scale. So we define the “rapidity” y of particle with u in whole range for both $u < c$ and $u > c$:

$$y = \ln \sqrt{\left| \frac{c + u}{c - u} \right|} \quad (33)$$

and find that both R and W can be expressed in a unified manner:

$$R = \tanh \left(\frac{y}{2} \right), W = \exp(-y) \quad (34)$$

which in turn are anticorrelated each other also in a unified way:

$$R = \frac{1 - W}{1 + W}, W = \frac{1 - R}{1 + R} \quad (35)$$

The mysteries of SR are now unveiled. The answer is ascribed to the monotonical increase of “hidden field of antiparticle state” and its phase evolution being opposite to

that of particle state essentially as shown by Eq.(15) versus (13). Though due to the condition $|\varphi/\chi| > 1$, the “hidden antiparticle field” χ is in subordinate position and is subjected to follow the “particle field” φ as shown in Eq.(13), it does impose an opposite tendency and enhance the inertial mass (\tilde{m}) of particle. In some sense, the time reading of clock accompanying φ is clockwise whereas that of χ is anticlockwise essentially. Though the time reading of a moving clock remains clockwise, it runs slower and slower with the enhancement of χ field.

Once the speed limit for a subluminal particle, the speed of light c , is broken through, a superluminal particle emerges with even more mysterious behavior as shown in Eqs. (3) and (9). Why they are so radically different from that of subluminal particle? The reason can be found from the difference between Eqs. (22) and (26) together with behavior of Weyl parameter. The normalization condition for Dirac particle $\int \rho_D d\vec{x}$ imposes stringent constraint on ξ_D and η_D such that $\lim_{u \rightarrow 0} \int \xi_D^+ \xi_D d\vec{x} = \lim_{u \rightarrow 0} \int \eta_D^+ \eta_D d\vec{x} = \frac{1}{2}$. By contrast, the normalization condition for superluminal particle $\int \rho d\vec{x} = 1$ imposes no constraint on ξ and η separately. The limiting behavior of $\lim_{u \rightarrow \infty} W = 1$ implies that both of them increase infinitely and approach to equal strength when $u \rightarrow \infty$. The fantastic consequence is their cancellation effect on the hidden antiparticle field $\chi = \frac{1}{\sqrt{2}}(\xi - \eta)$ to render $R \rightarrow 0$ and hence the tachyon energy $E = \tilde{m}c^2 \rightarrow 0$ while momentum $p = \tilde{m}u \rightarrow m_s c$.

Summary and discussion:

(a) Based on QM, a theory for superluminal particle is established. Being compatible with the theory of SR, it's actually a complement to SR. Now we realize that a particle can have speed u varying in the whole range $(0, \infty)$ with a universal constant c (the speed of light) as a singularity dividing superluminal particle (tachyon) from subluminal one. Most likely, the neutrino is just a tachyon with spin $1/2$.

(b) The crucial point is the cognition that “a particle is always not pure” [10-12] and there is no exception to neutrino. Now we realize that once if neutrino has some mass, no matter how tiny it is, two Weyl equations, (4) and (5), should be coupled together via some mass term while still respecting the maximum parity violation. Then Eq. (6) emerges almost as a unique possibility and an inevitable conclusion turns out to be that neutrino must be a superluminal particle with permanent helicity.

(c) Rewriting Eq. (11) in the form of four-component spinor equation, we find a

Dirac-type equation [7]:

$$i\hbar \frac{\partial}{\partial t} \psi_s = i\hbar \vec{\alpha} \cdot \nabla \psi_s + \beta_s m_s c^2 \psi_s, \quad (36)$$

$$\psi_s = \begin{pmatrix} \varphi \\ \chi \end{pmatrix}, \quad \alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}, \quad \beta_s = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}. \quad (37)$$

However, in comparison with Dirac Eq.(19), β_s is not a hermitian matrix. Now we realize that the violation of hermitian property is stemming from the violation of parity. Though a nonhermitian Hamiltonian is not allowed for a subluminal particle because it would lead to instability of solutions, it does work for a superluminal particle. Of four solutions for a same momentum of neutrino, two of them are eliminated due to the parity violation, corresponding to ν_R and $\bar{\nu}_L$ being forbidden strictly, and other two are stabilized, corresponding to physical realization of ν_L and $\bar{\nu}_R$. More importantly, Eq. (36) still preserves the invariance of basic symmetry (12).

(d) The parameter R defined in Eq.(29) could be understood as a measure of “impurity” of a particle being a superposition state of two hidden contradictory fields φ and χ ($|\varphi/\chi| > 1$). Though superficially, a free electron (neutrino) is always a particle with lepton quantum number $L = 1$, it does change intrinsically with its velocity. The larger R is, the larger mass(energy) and more instability it will have.

Similarly, the Weyl parameter W is the measure of “intrinsic instability of helicity” of a particle with superficial helicity $H = -1$, ($|\xi/\eta| > 1$). While an electron can turn its helicity to $H = 1$ when $|\eta/\xi| > 1$, a neutrino’s helicity is linked to lepton number L ($L = 1$, $H = -1$ whereas $L = -1$, $H = 1$) definitely. This difference is stemming from Eq.(26) versus (22). The common anticorrelation between R and W for both subluminal and superluminal particles implies that a high energy particle being more “impure” ($R \rightarrow 1$) will be more stable in helicity ($W \rightarrow 0$). On the contrary, a particle being unstable in helicity ($W \rightarrow 1$) corresponds to a “relatively pure” particle ($R \rightarrow 0$) with low energy. A prominent difference between a Dirac particle and neutrino is that for the former $\lim_{u \rightarrow 0} E = m_0 c^2$ whereas for the latter $\lim_{u \rightarrow \infty} E = 0$.

(e) When we talk about χ being the “hidden antiparticle amplitude” inside a particle and η being the “hidden right-handed helicity amplitude” inside a left-handed neutrino, we have to be cautious. As an example, for a high-energy electron with $R = 1/3$, can we

say that “it is composed of 75% (or 90%)electron ingredient and 25% (or 10%)antielectron (positron) ingredient”? No, we can’t. Because the χ field inside an electron is a probability amplitude and is in a subordinate position, no hidden opposite “charge” can be observed in a high-energy electron. The hidden χ field can only exhibit its implicit presence via the strange SR effects. Similarly, for a neutrino with $W = 1/3$, we can not say that “it is composed of 75% (or 90%) left-handed rotating state and 25% (or 10%)right-handed rotating state” because the neutrion is in 100% left-handed helicity state explicitly while both ξ and η fields enhance drastically and cancel each other considerably inside. Only in an antineutrino with $|\eta_c/\xi_c| > 1$, can η_c display itself as a right-handed rotating state, so does ξ_c follow accordingly.

Therefore, we should not interpret the “hidden probability amplitude ” too materialized in ordinary language. All fantastic behaviours of particle are due to the linear superposition and interference effect of fields between φ and χ (or ξ and η), not due to their intensity ($\varphi^+\varphi$ etc.). The existence of superluminal particle and its marvelous feature are new manifestations of subtlety of QM .In some sense,a particle is also a “Schrödinger’s cat”in microscopic scale and could be compared to the recent experimental verification of “macroscopic Schrödinger’s cat” [13] with its theoretical discussions [14,15].

(f) Consider a neutrino emitted in a supernova explosion. If being a tachyon, it will arrive at our earth earlier than the visible light by a time

$$\Delta T = \frac{D}{2c} \left(\frac{m_\nu c^2}{E} \right)^2 \quad (38)$$

with D and $m_\nu(E)$ being the distance of supernova and proper mass(energy) of neutrino repectively. On the other hand, the time span (ΔE) in the arrival of neutrino will reflect its spreading in energy (ΔE):

$$\Delta t \approx \frac{D}{c} \left(\frac{\Delta u}{c} \right) \approx \frac{D}{c} \left(\frac{m_\nu c^2}{E} \right)^2 \frac{\Delta E}{E} \quad (39)$$

$$\frac{\Delta t}{\Delta T} \approx 2 \frac{\Delta E}{E} \quad (40)$$

In Feb.23, 1987, a supernova explosion (SN 1987A) was observed. Two laboratories in Northern Hemisphere had detected 8 and 12 neutrino events within a time interval $\Delta t \approx 6$ and $13s$ respectively [16,17]. Since $D = 1.6 \times 10^5 ly$, using the data in [16], we

can estimate $\Delta u/c \sim 1.2 \times 10^{-12}$ and $(m_\nu c^2) = 44eV$ ($E \sim 32MeV$, $\Delta E \sim 20MeV$), which might be regarded as some upper bound for neutrino mass of all flavors. But more importantly, according to above estimation, laboratories in Southern Hemisphere could observe the light signal later than the neutrino events by a time lag $\Delta T \sim 5s$. It is regrettable for having no original record available and the theory of supernova explosion being too complicated for reliable conclusion could be drawn from so limited data.

(g) If neutrino is really a tachyon with its motion equation shown as Eq.(33) [i. e., Eq. (11) or (6)], then all the weak interaction processes in which neutrino participates need to be restudied. We hope new clues might be found for “neutrino oscillation” and the “missing puzzle of solar neutrino”. Especially, the mystery about dark matter in cosmos should be of the utmost concern.

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